Einstein's Hat: An Infinitely Non-Repeating Tessellating Tile

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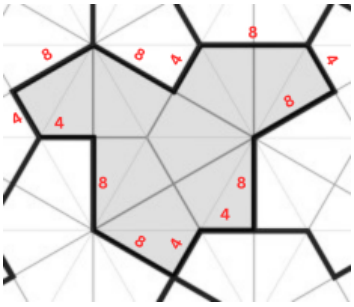
About the pattern

In March 2023, mathematicians D. Smith, J.S. Myers, C.S. Kaplan, and C. Goodman-Strauss solved a long-standing open question in geometry, namely, whether there is a geometric shape which can be tiled, or tessellated, with itself without ever forming any repeats or patterns. They came up with a 13-sided shape, which they call a "hat," because it vaguely looks like one.

This pattern is a rendering of that shape in crochet, allowing for the construction of arbitrarily-sized blocks made up out of these "hats" in a non-repeating pattern.

The hat tiles are built out of sides that are either 4 or 8 stitches long, and it can be helpful to think of parts of the pattern as "4's" or "8's". This diagram, adapted from Smith et al.'s paper, illustrates this.



The pattern begins at the bottom triangle, then builds the rectangle on top of the triangle, then extends out both directions; then completes the triangle on the left-hand and the trapezoid on the right-hand.

Materials needed

These tiles can be made with any combination of yarn weights and hook sizes. The "hats" in the photos are made with double knit acrylic yarn and a 3mm hook.

Abbreviations

This pattern uses US terminology. US sc = UK dc; US dc = UK tc.

2togdc combine two stitches into one dc

ch chain stitch

dc double crochet

dec decrease = combine two stitches into one sc

inc increase = two sc in one stitch

sc single crochet

Instructions

This pattern is worked in the flat. Every time you turn, always begin the next row in the same stitch unless explicitly noted otherwise.

(The bottom triangle)

Row 1 Make a starting loop, and ch6. Turn. (6 st total)

Row 2 In the third chain from hook 1dc, then 3dc, 2ch, turn. (6 st total)

Row 3 2togdc, 2dc, ch2, turn. (5 st total)

Row 4 1dc, 2togdc, ch2, turn. (4 st total)

Row 5 2togdc, ch1. (2 st total)

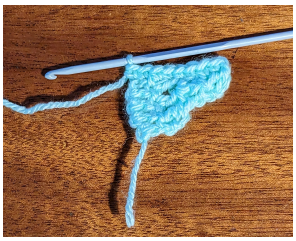
Figure 1: You should now have a triangle that looks like this:

(The rectangle)

Row 6 Turn 90 degrees, so that the long, straight side of the triangle is up (see photo).

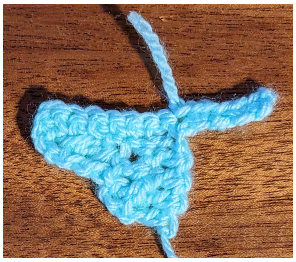
2sc in the side of each dc (8 stitches total), then ch5, turn. (13 st total)

Figure 2: After you've done the 8sc and before you do the ch5, your triangle should look like this:



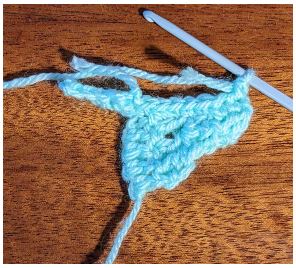
Row 7 3sc, slip stitch into next stitch, fasten off. (4 st total)

Figure 3: Your tile should now look like this:



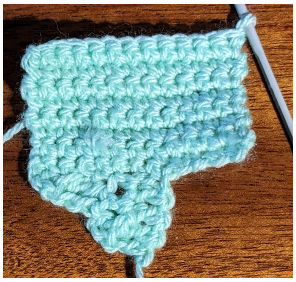
Row 8 Starting from the other end (see photo), begin with a standing sc, then 11sc, ch1, turn. (13 st total)

Figure 4: Begin row 8 here:



Rows 9–14 12sc, ch1, turn. (13 st total)

Figure 5: Your tile should now look like this:



(Extending the edges of the rectangle)

Row 15 12sc, ch5, turn. (17 st total)

Row 16 1inc, 15sc, 3ch, turn. (20 st total)

Row 17 1inc, 17 sc, 1inc, ch1, turn. (22 st total)

Row 18 1inc, 20sc, 3ch, turn. (25 st total)

Row 19 1inc, 22sc, 1inc, ch1 turn. (27 st total)

Figure 6: Your tile should now look like this:

(The left-hand triangle)

Row 20 1dec, 12sc, ch1, turn. (14 st total)

Row 21 11sc, 1dec, ch1, turn. (13 st total)

Row 22 1dec, 8sc, 1dec, ch1, turn. (11 st total)

Row 23 8sc, 1dec, ch1, turn. (10 st total)

Row 24 2dec, 3sc, 1dec, ch1, turn. (7 st total)

Row 25 1dec, 2sc, 1dec, ch1, turn. (5 st total)

Row 26 2dec, ch1, turn. (3 st total)

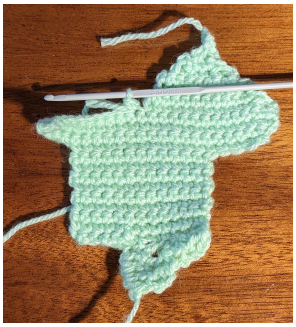
Row 27 1dec, finish off. (1st total)

Figure 7: Your tile should now look like this:

(The right-hand trapezoid)

Row 28 Starting next to the last stitch in row 20, do a standing sc to begin, then 11sc, ch3, turn. (15 st total)

Figure 8: This is how it will look at the start of row 27:



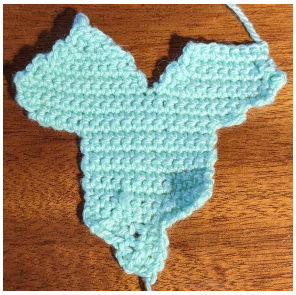
Row 29 1dec, 10sc, 1dec, ch1, turn. (13 st total)

Row 30 1dec, 8sc, 1dec, ch1, turn. (11 st total)

Row 31 1dec, 6sc, 1dec, ch1, turn. (9 st total)

Row 32 8sc, finish off. (8 st total)

Figure 9: This is how your finished tile should look



Attach the pieces using either 4 or 8 sc depending on whether you're joining a 4 or an 8 side. In order to tessellate the pieces, some of them will have to be flipped so that their "back side" is facing up. Determining how the pieces tessallate with each other can be complex; we recommend looking at the diagrams in Smith et al.'s original paper for assistance. This paper is freely available on arXiv.org: `https://arxiv.org/abs/2303.10798`.

License

Version history

V. 1 (June 19, 2023); v. 1.1 (June 20, 2023), corrected number of stitches in row 17, and two typos in the opening section.